Projectile Motion Physics Problems

Key Points:

1. Objects that are projected from, and land on the same horizontal surface will have a vertically symmetrical path.
2. The time it takes from an object to be projected and land is called the time of flight. This depends on the initial velocity of the projectile and the angle of projection.
3. When the projectile reaches a vertical velocity of zero, this is the maximum height of the projectile and then gravity will take over and accelerate the object downward.
4. The horizontal displacement of the projectile is called the range of the projectile and depends on the initial velocity of the object.

Key Terms:

Trajectory: The path of a body as it travels through space

Projectile Motion:

Projectile motion is a form of motion where an object moves in a bilaterally symmetrical, parabolic path. The path that the object follows is called its trajectory. Projectile motion only occurs when there is one force applied at the beginning on the trajectory, after which the only interference is from gravity

Initial Velocity:

The initial velocity can be expressed as x components and y components:

$u_x = u_0 \cdot \cos\theta$

$u_y = u_0 \cdot \sin\theta$

In this equation, u_0 stands for initial velocity magnitude and θ refers to projectile angle.

Time of Flight:

The time of flight of a projectile motion is the time from when the object is projected to the time it reaches the surface.
t depends on the initial velocity magnitude and the angle of the projectile:

$t = (2 \cdot u_y / g)$

$t = (2 \cdot u \cdot \sin\theta) / g$

Acceleration:

In projectile motion, there is no acceleration in the horizontal direction. The acceleration, a, in the vertical direction is just due to gravity, also known as free fall:

$a_x = 0$

$a_y = -g$

Velocity:

The horizontal velocity remains constant, but the vertical velocity varies linearly, because the acceleration is constant.
At any time, t, the velocity is:

$u_x = u_0 \cdot \cos\theta$

$u_y = u_0 \cdot \sin\theta - (g \cdot t)$

You can also use the Pythagorean Theorem to find velocity:

$u = \sqrt{(u^2{}_x + u^2{}_y)}$

Displacement:

At time t, the displacement components are:

$x = u \cdot t \cdot \cos\theta$

$y = u \cdot t \cdot \sin\theta - 0.5gt^2$

The equation for the magnitude of the displacement is $\Delta r = \sqrt{(x^2 + y^2)}$

Parabolic Trajectory:

We can use the displacement equations in the x and y direction to obtain an equation for the parabolic form of a projectile motion:

$y = \tan\theta \cdot x - (g/(2 \cdot u^2 \cdot \cos^2\theta) \cdot x^2$

Maximum Height:

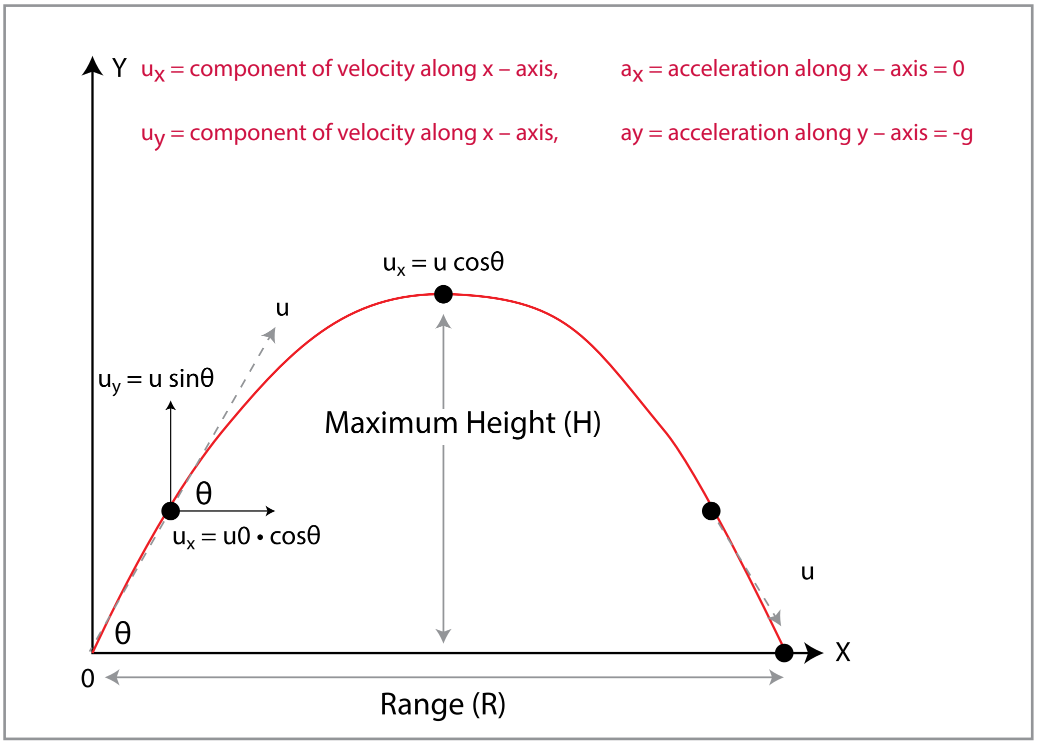
The maximum height is reached when $u_y = 0$. Using this we can rearrange the velocity equation to find the time it will take for the object to reach maximum height

$t_h = (u \cdot \sin\theta) / g$

where t_h stands for the time it takes to reach maximum height. From the displacement equation we can find the maximum height

$h = (u^2 \cdot \sin^2\theta) / (2 \cdot g)$

The Science of Projectile Motion



Example Problems:

A small rocket is launched at a 30 degree angle at 25 m/s.

1) What is the maximum height reached by the rocket?

The formulas we need to solve this problem are the following:

- $u_x = u_0 \cdot \cos\theta$
- $u_y = u_0 \cdot \sin\theta - (g \cdot t)$
- $g = 9.8\ m/s^2$

At its maximum height, $u_y = u_0 \cdot \sin\theta - (g \cdot t) = 0$

First, solving for t:

$t = (u_0 \cdot \sin\theta) / g$

$t = (25 \cdot \sin 30) / 9.8$

$t = 1.28$ seconds

Now solve for y:

$y = u \cdot t \cdot \sin\theta - 0.5gt^2$

$y = (25 \cdot 1.28 \cdot \sin 30) - (0.5 \cdot 9.8 \cdot 1.28^2)$

$y\ (max) = 7.97\ m$

Example Problems:

2) What is the total time in flight of the rocket?

The formulas we need to solve this problem are the following:

- $u_x = u_0 \cdot \cos\theta$
- $u_y = u_0 \cdot \sin\theta - (g \cdot t)$

t at the time of launch is t_1 which = 0
t at the time the rocket lands (ground) is t_2. At t_2, y = 0.

Therefore, if $y = u \cdot t \cdot \sin\theta - 0.5gt^2$:

$y = t(u \cdot \sin\theta - 0.5gt)$	or $y = 2(u \cdot \sin\theta) - gt$
At t_2, y = 0,	so $2(u \cdot \sin\theta) - gt = 0$

$t = 2(u \cdot \sin\theta) / g$
$t_2 = 2(25 \cdot \sin 30) / 9.8$
$t_2 = 2.55$ sec
so time of flight = $t_2 - t_1 = 2.55 - 0 = 2.55$ seconds

3) What is the horizontal range (distance covered) of the rocket?

The formulas we need to solve this problem are the following:

- $x = u \cdot t \cdot \cos\theta$

So in this case, we are looking for x at $t_2 - t_1$ (which is 2.55 seconds from part b)
$x = 25 \cdot 2.55 \cdot \cos 30 = 32.3$ meters

4) What is the magnitude of the velocity of the object just before it hits the ground?

The formulas we need to solve this problem are the following:

- $u_x = u0 \cdot \cos\theta$
- $u_y = u0 \cdot \sin\theta - (g \cdot t)$
- $u = \sqrt{}(u^2\, x + u^2\, y)$

We are solving for the velocity components at $t_2 = 2.55$ seconds

$u_x = u0 \cdot \cos\theta$	$u_x = 25 (\cos 30) = 23.5$ m/s
$u_y = u0 \cdot \sin\theta - (g \cdot t)$	$u_y = 25 (\sin 30) - (9.8 \cdot 2.55) = - 12.5$ m/s
$u = \sqrt{}(u^2\, x + u^2\, y)$	$u = \sqrt{}(23.5)^2 + (-12.5)^2 = 26.6$ m/s